

Ballistics Research Report 168

A PRELIMINARY STUDY OF THE MECHANISM OF WATER-  
ENTRY WHIP OF 20° CONE NOSED MODELS

Prepared by:  
John O. Gurney, Jr.

ABSTRACT: Preliminary experimental data are reported on the whip behavior of 20-degree cone nosed models with variable center of gravity and variable transverse moment of inertia launched at 45 degrees and at 105-135 feet per second. The results are compared with fundamental ideas and it is concluded that there is a high sensitivity of whip to initial pitch and that there may be a force distribution on the nose producing the effect of a couple during nose penetration.

The data reported are incomplete; for this reason, all results and conclusions are not to be taken as final.

NOLTR 66-148

Ballistics Research Report 168

A PRELIMINARY STUDY OF THE MECHANISM OF WATER-  
ENTRY WHIP OF 20° CONE NOSED MODELS

Prepared by:  
John O. Gurney, Jr.

ABSTRACT: Preliminary experimental data are reported on the whip behavior of 20-degree cone nosed models with variable center of gravity and variable transverse moment of inertia launched at 45 degrees and at 105-135 feet per second. The results are compared with fundamental ideas and it is concluded that there is a high sensitivity of whip to initial pitch and that there may be a force distribution on the nose producing the effect of a couple during nose penetration.

The data reported are incomplete; for this reason, all results and conclusions are not to be taken as final.

U. S. NAVAL ORDNANCE LABORATORY  
WHITE OAK, MARYLAND

NOLTR 66-148

1 August 1966

A PRELIMINARY STUDY OF THE MECHANISM OF WATER-ENTRY WHIP OF  
20° CONE NOSED MODELS

The results reported are from a continuation of the effort at the Naval Ordnance Laboratory to develop low drag water entry configurations.

The author thanks Douglas F. Newell for the setup and operation of the electronics, and John L. Baldwin, Jr. for assisting in the experimental procedure.

E. F. SCHREITER  
Captain, USN  
Commander

  
W. CARSON LYONS, JR.  
By direction

# NOLTR 66-148

## CONTENTS

	Page
INTRODUCTION. . . . .	1
DISCUSSION OF EXPERIMENTAL APPARATUS. . . . .	3
THE FIRING PROGRAM. . . . .	6
DISCUSSION OF RESULTS . . . . .	13
SUGGESTIONS FOR FURTHER INVESTIGATION . . . . .	19
REFERENCES. . . . .	20

## ILLUSTRATIONS

Figure	Title	Page
1	Missile Free Body Diagram . . . . .	1
2	Typical Whip Record for Ogive Nose. . . . .	2
3	Experimental Schematic. . . . .	4
4	Whip Record . . . . .	5
5	Side View Photograph. . . . .	5
6	Oscilloscope Trace. . . . .	5
7	Whip Record Calibration . . . . .	5
8	$\Delta\theta/\Delta T$ vs Depth Curve, Model A . . . . .	8
9	$\Delta\theta/\Delta T$ vs Depth Curve, Model B . . . . .	9
10	$\Delta\theta/\Delta T$ vs Depth Curve, Model C . . . . .	10
11	$\Delta\theta/\Delta T$ vs Depth Curve, Model D . . . . .	11
12	$\Delta\theta/\Delta T$ vs Depth Curve, Model E . . . . .	12
13	Whip vs Pitch (Estimate), Model E . . . . .	13
14	Whip vs Initial Pitch, Model E. . . . .	14
15	Whip vs Pitch (Estimate), Models A & B. . . . .	15
16	Whip vs Pitch (Estimate), Model C . . . . .	15
17	Whip vs Pitch (Estimate), Model D . . . . .	16
18	Plot of Eq. (10) . . . . .	17
19	Whip vs $I_{cg}$ . . . . .	19

## TABLES

Table	Title	Page
1	Model Configurations. . . . .	6
2	Parameter Errors. . . . .	7

LIST OF SYMBOLS

A	instantaneous effective pressure area
$\bar{A}$	effective time average of A
C	constant
CG	distance from center of gravity to cone base
D	diameter of model
F	effective time average lift force
I	transverse moment of inertia of model about its center of gravity
L	distance from center of gravity to instantaneous center of pressure
$\bar{L}$	effective time average of L
M	moment about center of gravity
P	pressure
R	instantaneous total pressure force
$\bar{R}$	effective time average R
S	axial depth of penetration beneath water surface
V	velocity
$V_0$	initial (water contact) velocity
X	L - CG
$\theta$	trajectory angle
$\theta_0$	initial (water contact) trajectory angle
$\Delta\theta$	change in model axis pitch orientation
$\Delta\theta/\Delta T$	time rate of $\Delta\theta$
$\varphi$	angle between outward normal of a surface and the velocity vector
$\omega$	$\Delta\theta/\Delta T$

# INTRODUCTION

The classical definition of the term "whip" is the change in angular velocity about an axis perpendicular to the plane of motion of a missile during the submergence of the nose (figure 1). This change in  $\Delta\theta/\Delta t$  is caused by an unbalance of forces due to flow forming when a missile enters at other than 90 degrees, which should, according to a simplified theory and previous experimental

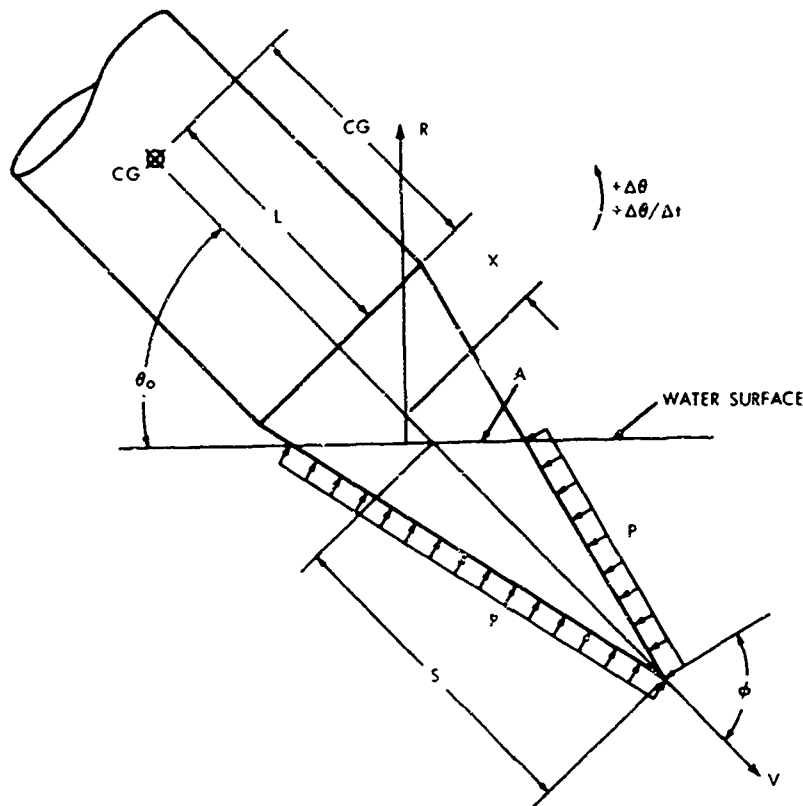


Fig. 1. Missile Free Body Diagram

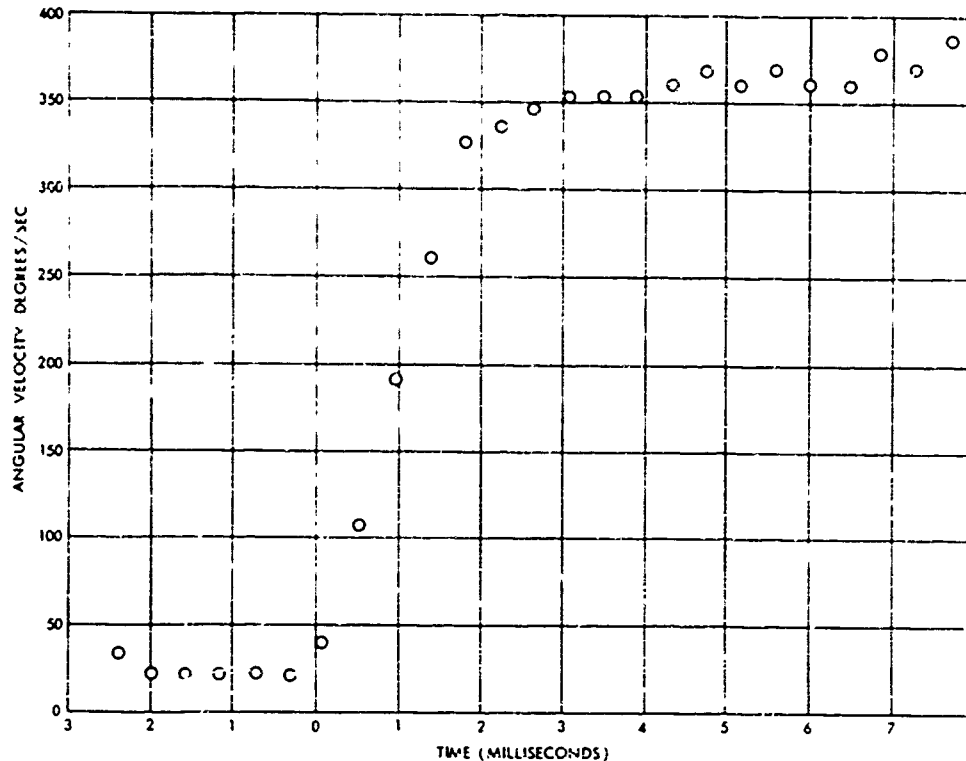


Fig. 2. Typical "Whip" Record for Ogive Nose

results (figure 2), revert to near zero after the nose submerges, reference 1.

The missile nose striking a water surface experiences at first a shock force of short duration followed by a flow forming stage. The former has been shown to have little effect on the missile motion, references 2 and 3. The latter is hypothesized to obey some sort of similarity law. Birkhoff (ref. 2, 3 and 4) assumed a constant pressure to act over the wetted portion (ignoring splash) of the nose.

$$p = C_1 \rho V^2 \cos^2 \theta \quad (1)$$

To simplify the analysis, assume that a symmetrical cone enters at angle  $\theta$  and zero pitch and that velocity, trajectory angle and pitch remain constant throughout the flow forming stage. Then Equation (1) becomes

$$p = C_2 \rho V_0^2 \quad (2)$$

This constant pressure over the wetted surface results in a vertical force  $R$  acting at a center of pressure which lies on the plane of intersection of the cone with the water surface (figure 1).

The instantaneous whip  $w_s - w_o$  at any depth  $s$  can be non-dimensionalized as follows:

The moment equation about the center of gravity is:

$$M = pA \sin \theta L = C_2 \rho V_o^2 A L \quad (3)$$

$$C_2 \rho V_o^2 A L = I \, dw/dt = I \frac{dw}{ds} \frac{ds}{dt} \quad (4)$$

$$\int_0^s C_2 \rho V_o^2 A(s) L(s) ds = \int_{w_o}^{w_s} I dw = I(w_s - w_o) \quad (5)$$

Now  $A(s)$  and  $L(s)$  can be extracted from the integral by replacing them with an average  $\bar{A}$  and  $\bar{L}$  which would cause the same effect as  $A(s)$  and  $L(s)$  integrated from 0 to  $s$ . Thus, since  $\bar{A}$ ,  $\bar{L}$  and  $s$  are functions of  $D$ ,

$$C_3 \rho V_o D^4 = I \Delta w \quad (6)$$

$$C_3 = I \Delta w / \rho V_o D^4 \quad (7)$$

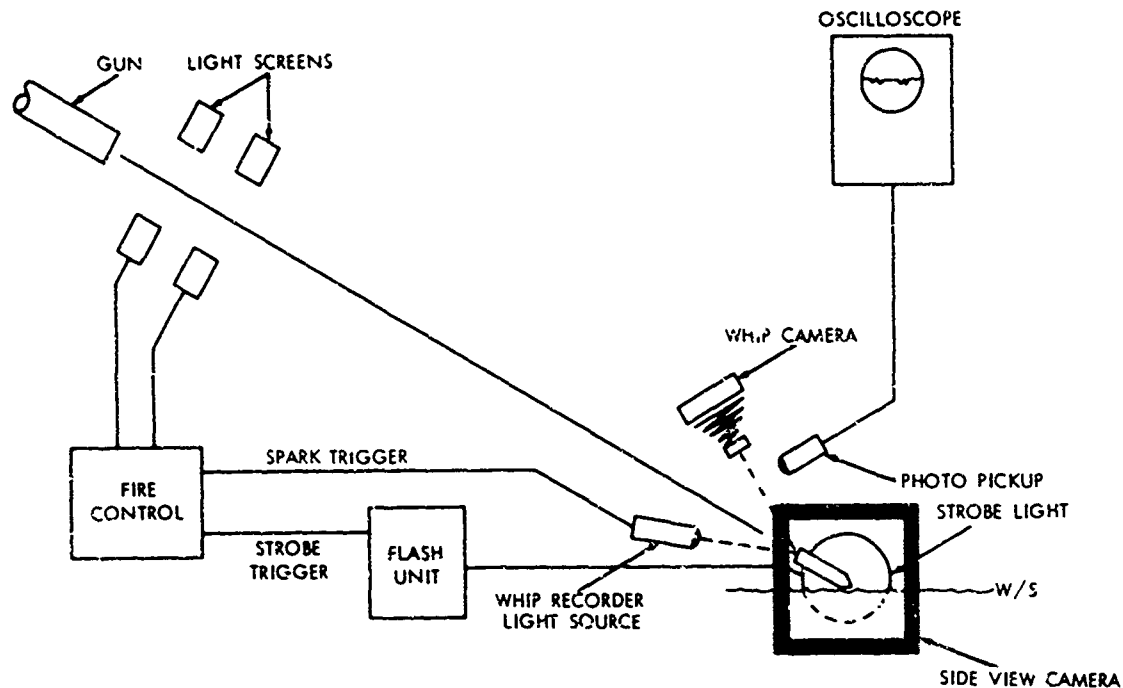
The linear variation of whip with  $V_o$  implied in Equation (7) has been verified in experiment (ref. 5).

Taking the above results as a starting point, an experimental whip study was begun at the Naval Ordnance Laboratory using an optical whip recorder after the design of B.H. Rule (ref. 6). The results included here are for 1.5-inch-diameter models with 20-degree cone noses launched at 45 degrees from vertical.

#### DISCUSSION OF EXPERIMENTAL APPARATUS

The operation of the whip recorder is similar to that discussed in reference 6; therefore, only the system external to the basic





**Fig. 3. Experimental Schematic**

whip recorder will be described (figure 3). One innovation, an auxiliary spark source (see ref. 7), was installed in the light source housing to permit a precise zero time determination. This spark source is flashed once while the model is tracing its whip record, producing a dot on the whip camera film next to one of the dots on the whip trace, figure 4. Both dots are then known to have occurred at about the same time, and this provides a link between the relative time of the whip trace dots and the real time of the auxiliary spark. The firing of the spark source, as well as other events, is sequenced by a digital fire control unit, reference 7, which receives its input when the model passes through a set of light screens near the gun muzzle. The light screens also serve as velocity pickups. Just before water impact, the model passes between a strobe light and an open-shutter camera. The strobe flashes three times, placing three shadow images on the side camera film, figure 5. This photo is used in the zero time correlation and in a pitch measurement. All events are correlated in time through an oscilloscope which records inputs from a photo pickup. This pickup senses light pulses from the chopped whip recorder light, the auxiliary spark and the strobe light, figure 6.

In lieu of a thorough error analysis, a few remarks on data interpretation will be made. While the distance between the dots on the whip trace (figure 4) can be reduced to 1/100 inch (which is equivalent to .002 degree model rotation), this is true only on the latter portion of the trace. Since a model is rotating

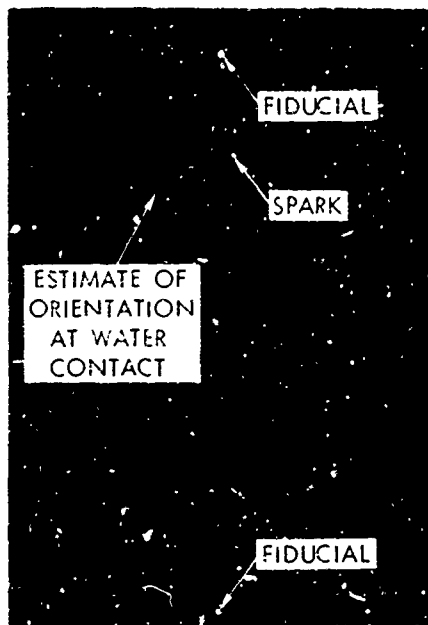


FIG. 4 WHIP RECORD



FIG. 5 SIDE VIEW PHOTOGRAPH

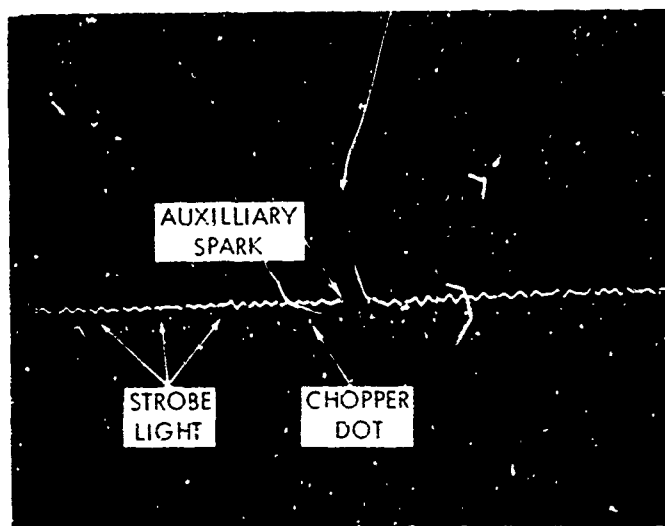


FIG. 6 OSCILLOSCOPE TRACE

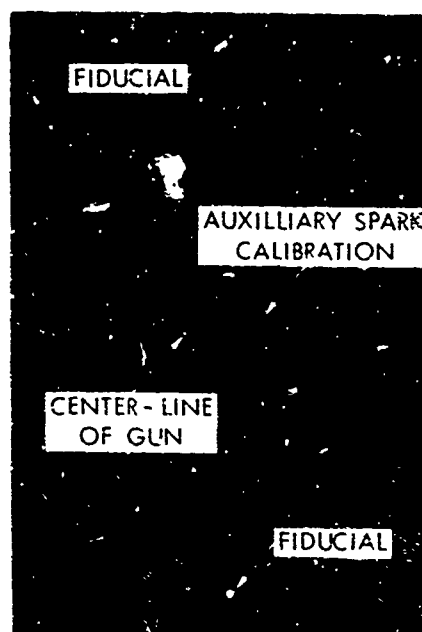


FIG. 7 WHIP RECORD CALIBRATION

# NOLTR 66-148

comparatively slowly at and just after water impact, any chopper dots recorded in this region pile on top of one another and cannot be reduced. The value of  $(\Delta\theta/\Delta T)_0$  is not known and  $\theta_0$  is known only to about 1/10 degree. Thus, curves of  $\Delta\theta/\Delta T - (\Delta\theta/\Delta T)_0$  or  $\theta - \theta_0$  vs time cannot be plotted accurately. For this report the whip trace was stepwise differentiated and  $\Delta\theta/\Delta T$  vs depth curves were plotted.

In the course of the firing program, a need for recording pitch just before water contact was realized. This measurement was attempted using the whip record. For a zero angle reference, a stationary mirror aligned perpendicular to the gun bore, using an autocollimator, was mounted in the whip recorder's field of view. A single exposure of the chopper dot was then taken (figure 7). An estimate of initial pitch was then made by comparing succeeding records (figure 4) with the calibration record, assuming that the gun bore-whip recorder alignment is always fixed and that all firings of a given model at the same velocity follow the same gravity trajectory. In truth, the validity of these assumptions, especially the latter, was somewhat in doubt. Pitch measurements on the last few shots of the program were made using the side camera mounted with a 360-mm lens as a check on the whip record method. The 4x5 negatives (figure 5) were read for pitch and trajectory angle on a 10X optical comparator.

Incidentally, the whip records and scope traces were read on a B&L toolmaker's microscope with .0001 moving micrometer stage. This scope gave sufficient precision.

## THE FIRING PROGRAM

While 1.5-inch-diameter models with several different nose shapes have been launched in the past year, only those launchings pertinent to the examination of Equation (7) will be discussed. These configurations are 20-degree cone models with variable  $I_{cg}$  and C.G.

Table 1. Model Configurations

Model Type	Wt. lb.	C G in.	Velocity ft/sec	$I_{cg}$ lb/in <sup>2</sup>	No. of Shots
A	.96	3.48	135	8.50	2
B	1.04	3.67	135-136	9.71	2
C	1.33	.46	132-177	10.4	2
D	2.07	3.14	101-105	8.15	3
E	2.12	3.14	101-108	22.1	11

Nose shape = 20° cone  
Water-entry angle = 45°  
Diameter = 1.5 inch

# NOLTR 66-148

Admittedly, the variation in weight (an extraneous variable) is quite large. For the most part, the models were not designed for constant weight. However, this variable is seen to have a small effect on whip, since the only time of interest is during the brief period of nose entry; and the only measurements taken are, indirectly, the lift and drag forces on the nose. If it can be assured that the model has great enough mass that any slowdown during this interval is small, then one can assume constant velocity at water entry and, therefore, all other things being equal, the transient flow field will reproduce. A calculation based on drag coefficients derived from the virtual mass approximation to water entry (ref. 8) yields the following slowdown velocities for  $V_0 = 105$  fps:

Missile Mass =	.96 lbm	$\Delta V = -1.5$ fps
	1.32 lbm	-1.0 fps
	2.12 lbm	- .5 fps

If, in the future, measurements are taken of deceleration, or, if for scaling purposes, models of low C/S density are used, then mass must be controlled.

The control of operating parameters and the recording of data underwent improvement through the year. Listed are the estimated precision limits for data taken in the last series of shots.

Table 2. Parameter Errors

<u>Quantity</u>	<u>Estimate of Precision</u>
Model mass	$\pm 1$ g
Velocity of model	$\pm 2$ fps
$I_{cg}$ of model	$\pm .3$ lb/in <sup>2</sup>
CG of model	$\pm 1/64$ inch along axis
	$\pm .002$ inch radially
$\Delta\theta/\Delta T$	$\pm 4$ deg/sec
Pitch at W/E by	
Side view picture	$\pm .1$ degree
Whip trace	$\pm .2$ degree
$(\Delta\theta/\Delta T)_0$	$< 20$ deg/sec
Whip curve zero time	$\pm .1$ inch

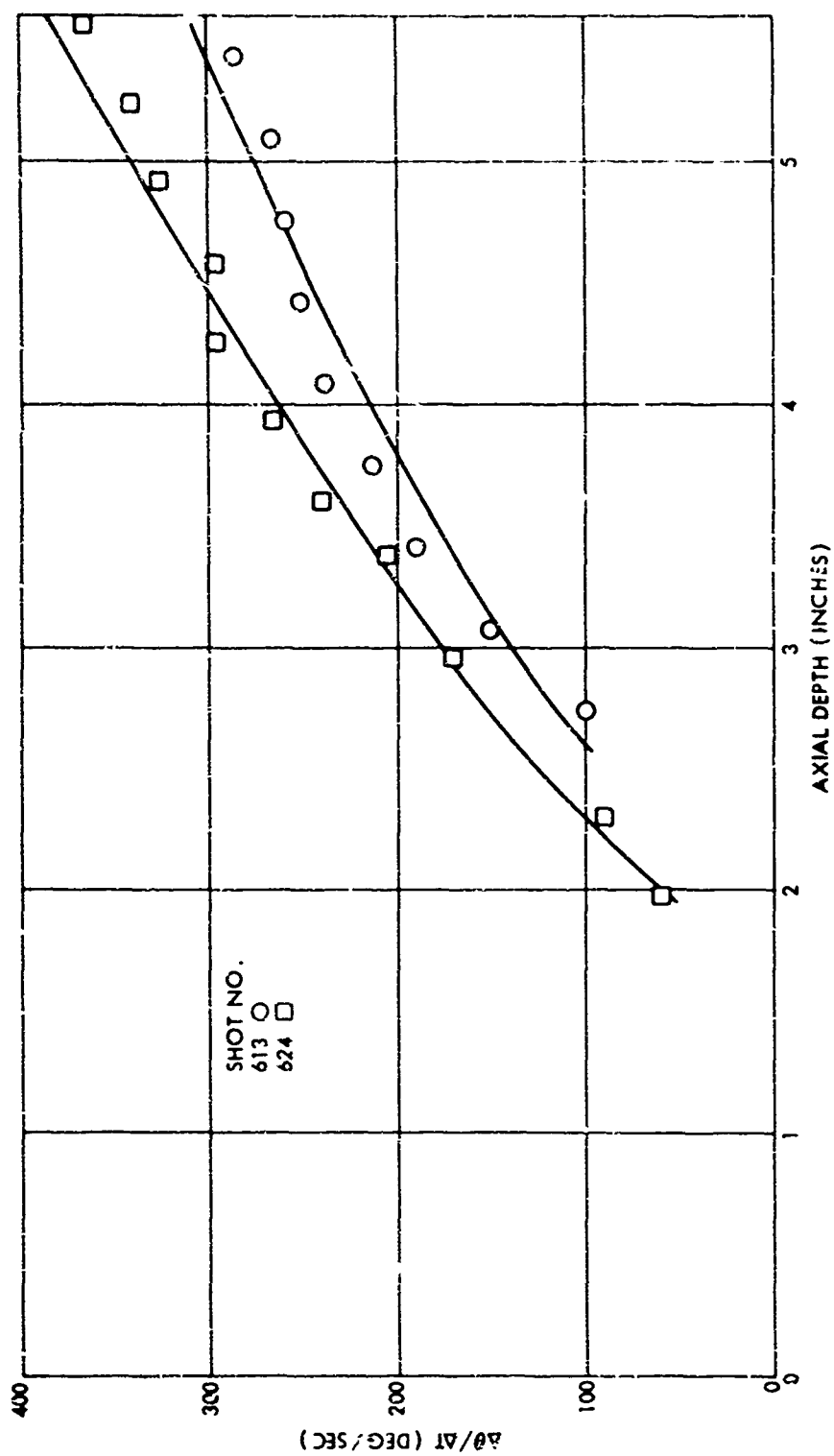


FIG. 8  $\Delta\theta/\Delta t$  VS DEPTH CURVE, MODEL -A

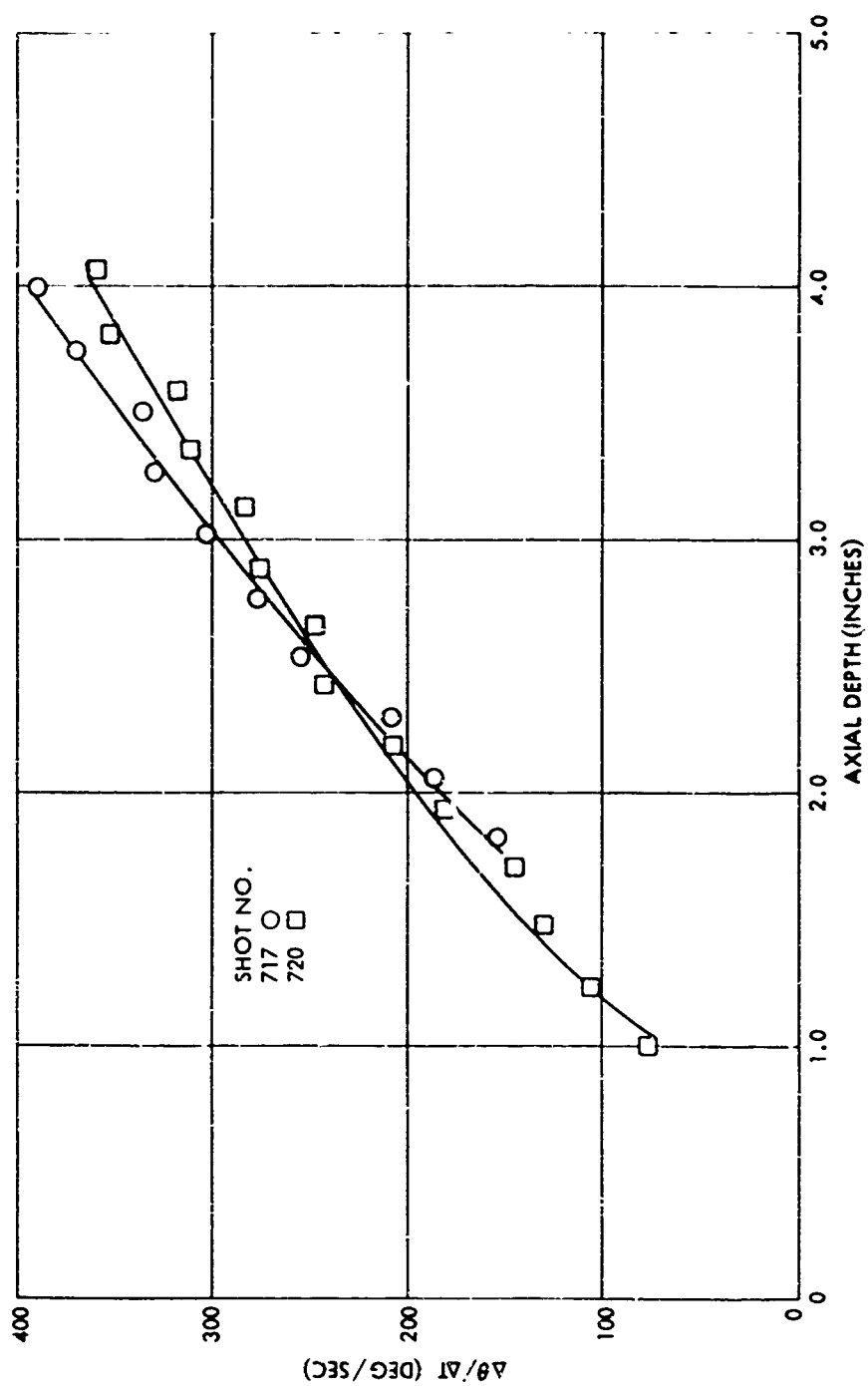


FIG. 9  $\Delta\theta/\Delta t$  VS DEPTH CURVE, MODEL - B

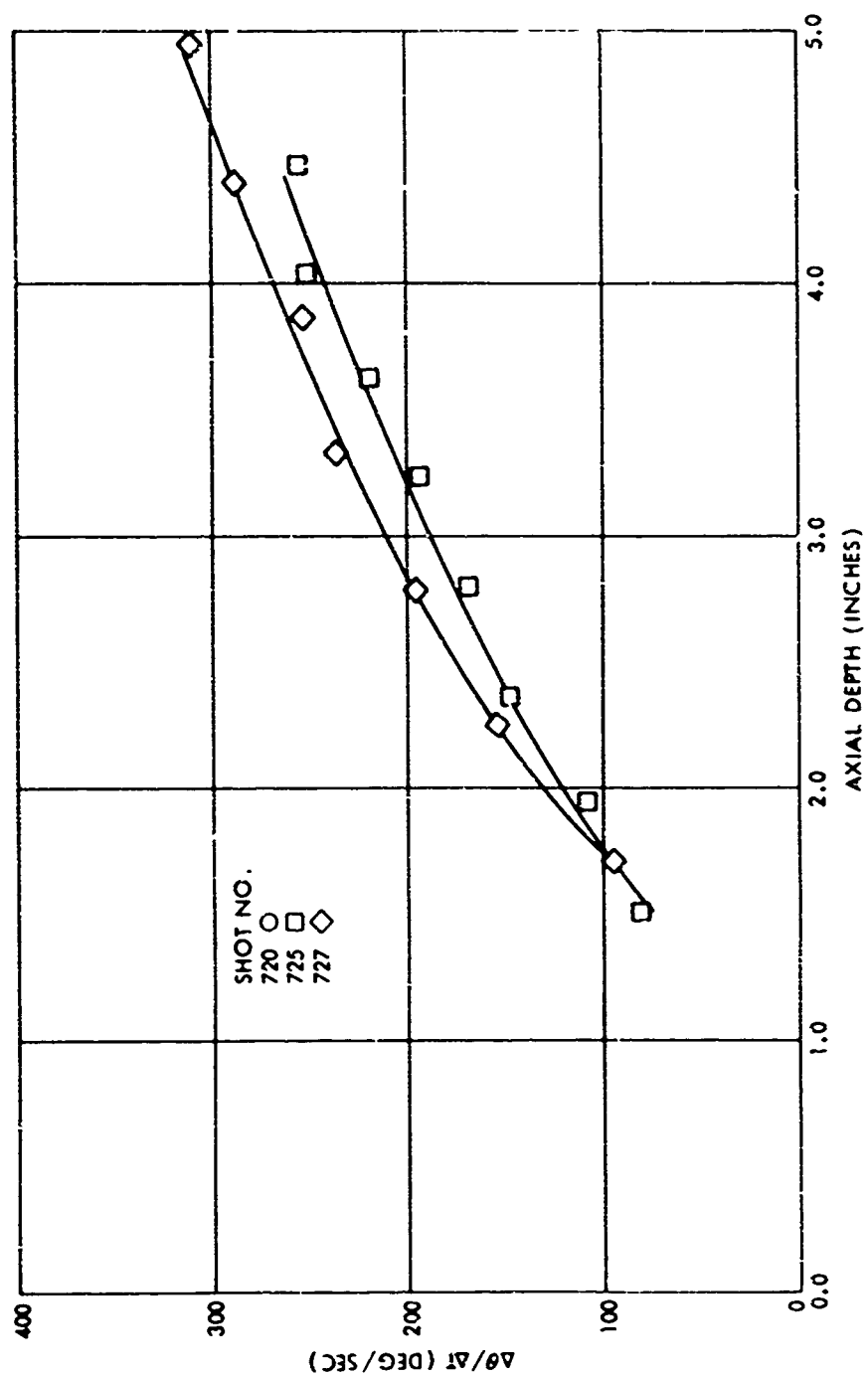


FIG. 10  $\Delta\theta/\Delta t$  VS DEPTH CURVE, MODEL -C

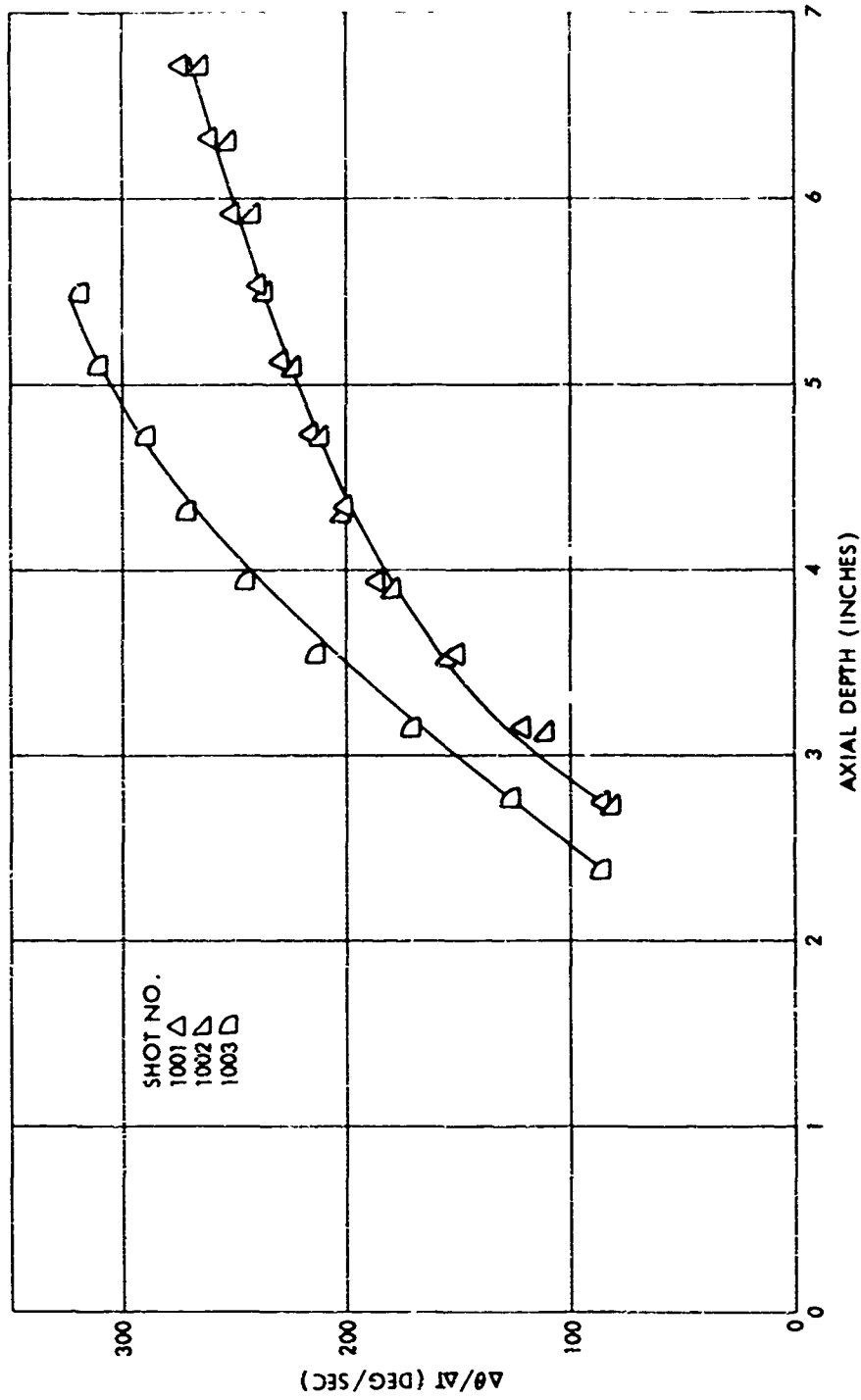


FIG. 11  $\Delta\theta/\Delta t$  VS DEPTH CURVE, MODEL-D



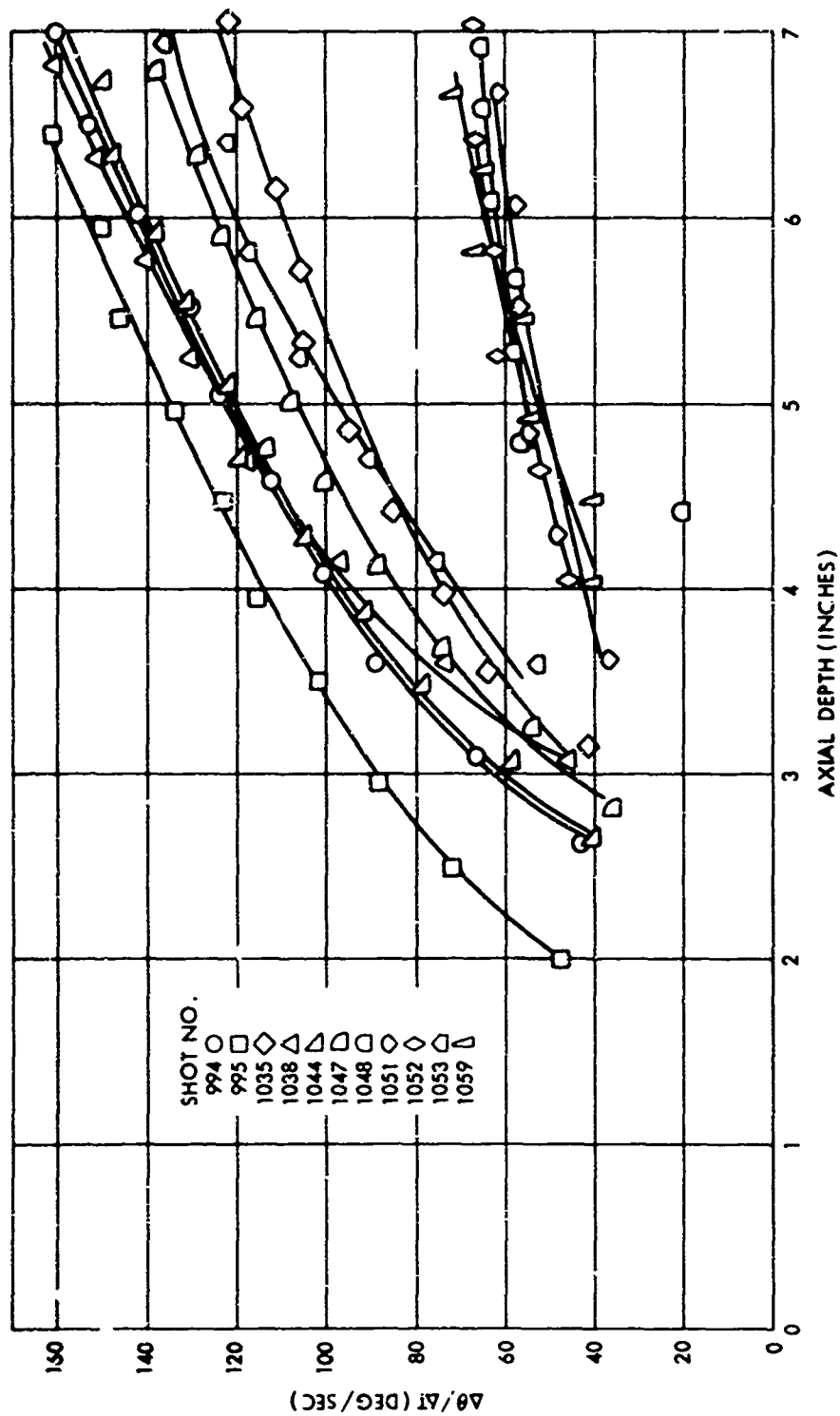


FIG. 12  $\Delta\theta/\Delta t$  VS DEPTH CURVE, MODEL-E

## DISCUSSION OF RESULTS

The whip record traces are presented in the form of  $\Delta\theta/\Delta T$  vs axial depth curves for models A, B, C, D and E (figures 8, 9, 10, 11 and 12), as discussed in the second section, "Discussion of Experimental Apparatus." The choice of depth rather than time for independent variable is perhaps more meaningful in light of Equation (7),  $C = I\Delta\omega/\rho V_0 d^4$ . The line of reasoning behind this equation concludes that whip varies linearly with entry velocity. Hence, the angle change of a missile is independent of velocity and depends only on depth. In any case, whip is taken as  $\Delta\theta/\Delta T - (\Delta\theta/\Delta T)_0$  at a certain depth of the nose. Now, the curves shown do not agree with the constant pressure assumption in that there is no leveling off of  $\Delta\theta/\Delta T$  after the nose submerges (at 5-inch axial depth). Also, all that can be said about  $(\Delta\theta/\Delta T)_0$  is that it is less than 20 degrees per second in most cases. Therefore, the value of  $\Delta\theta/\Delta T$  at depth 5 inches was used in the correlation of whip with parameters.

The initial conditions ( $V_0$ , pitch, and  $(\Delta\theta/\Delta T)_0$ ) have an obvious effect on  $\Delta\theta/\Delta T$  curves. In figure 12 the curves seem to fan out as depth increases. As the velocities of these shots were almost identical,  $105 \pm 3$  feet per second, and  $(\Delta\theta/\Delta T)_0$  is comparatively small, the fan-out effect appears as a pitch sensitivity, and this is supported in figures 13 and 14 which indicate

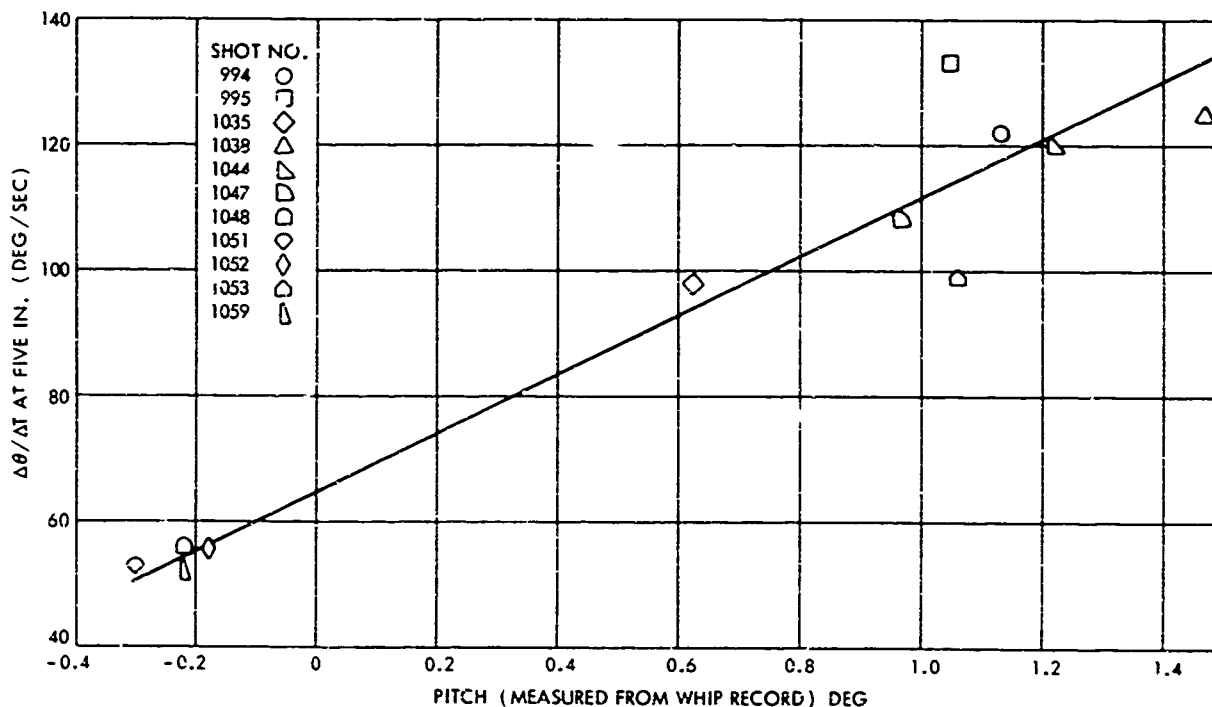


Fig. 13. Whip vs Pitch (Estimate), Model E

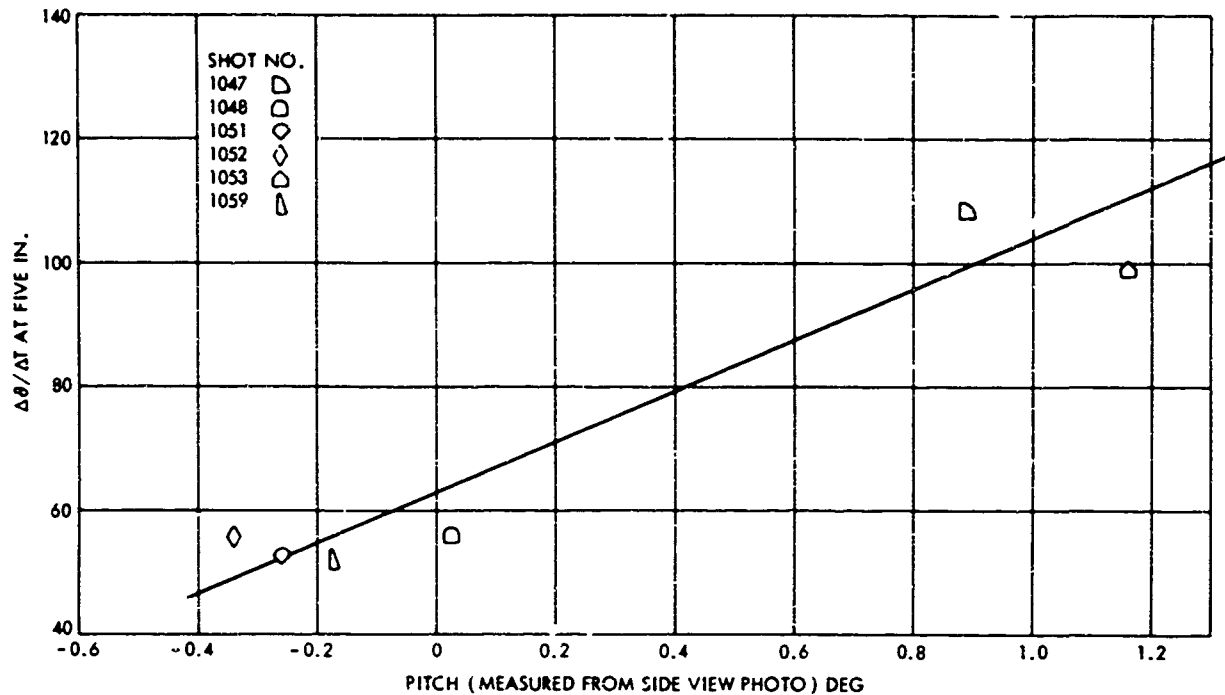


Fig. 14. Whip vs Initial Pitch, Model E

a linear pitch sensitivity, defined as the slope of the whip vs pitch curve. This is similar to Waugh's results, reference 9, which indicate linear pitch sensitivities for full-scale ogive-nosed missiles. A comparison of modeled pitch sensitivities of model D and Waugh's 3.5-inch diameter ogive-nosed missile entering at 20 degrees from the water surface reveals that the cone-nosed missile may be as sensitive to pitch as a slender ogive-nosed missile: 18.1 deg/sec (whip) per degree pitch vs 13.6 deg/sec-degree. The basis for this comparison is that the 3.5-inch diameter ogive model is roughly scaled by model D, using dynamic and Froude scaling laws (ref. 3). Incidentally, the pitch of the last six shots of model E was measured from a large image on the side view camera, and whip vs pitch for these shots is plotted in figure 14. Figure 13 is a plot of whip vs pitch of all shots of model E, including the last six. However, the values of pitch used in this plot were determined using the whip recorder, as discussed in the second section, "Discussion of Experimental Apparatus." While there is some discrepancy in individual points between figures 13 and 14, the straight line approximations are almost identical. Plots of whip vs pitch (measured by whip record) for shots A, B, C and D are shown in figures 15, 16 and 17. Assuming a linear pitch sensitivity indicated by the results of model E, straight lines were drawn through the points to extrapolate a value of whip for zero pitch.

NOLTR 66-148

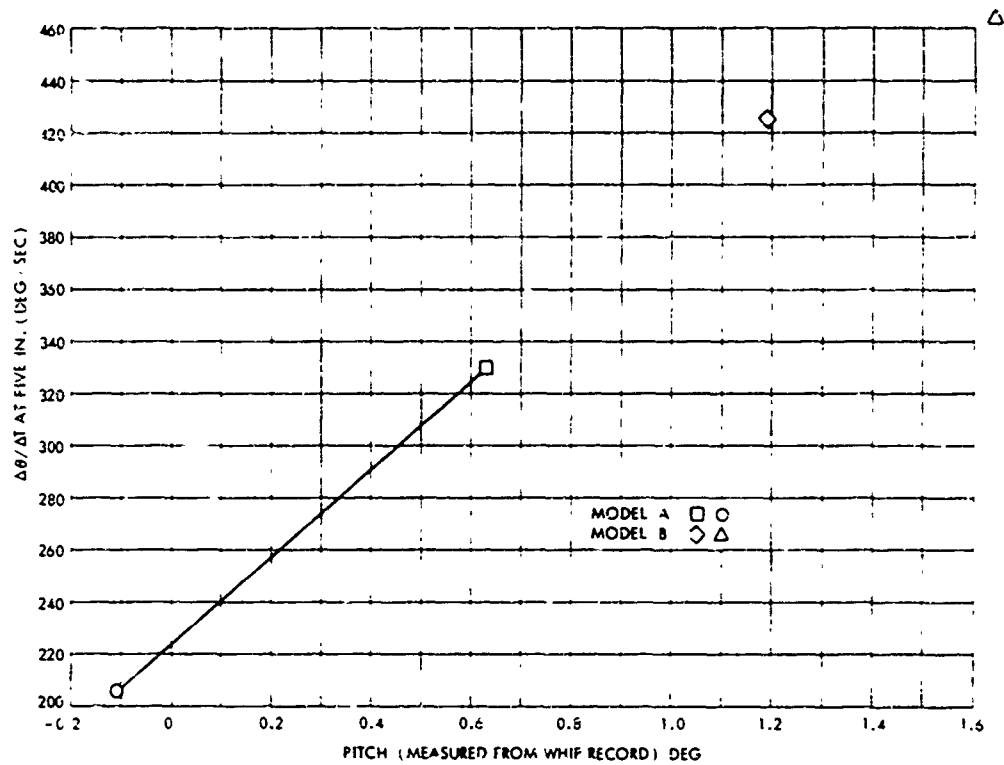


Fig. 15. Whip vs Pitch (Estimate), Models A and B

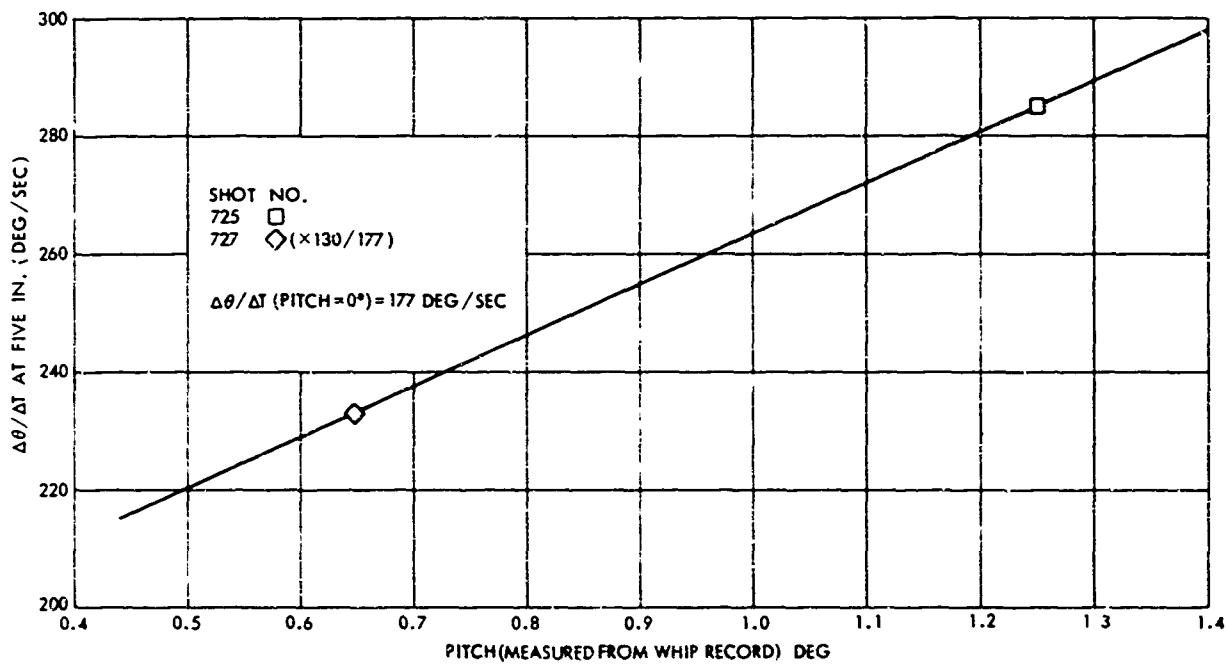


Fig. 16. Whip vs Pitch (Estimate), Model C

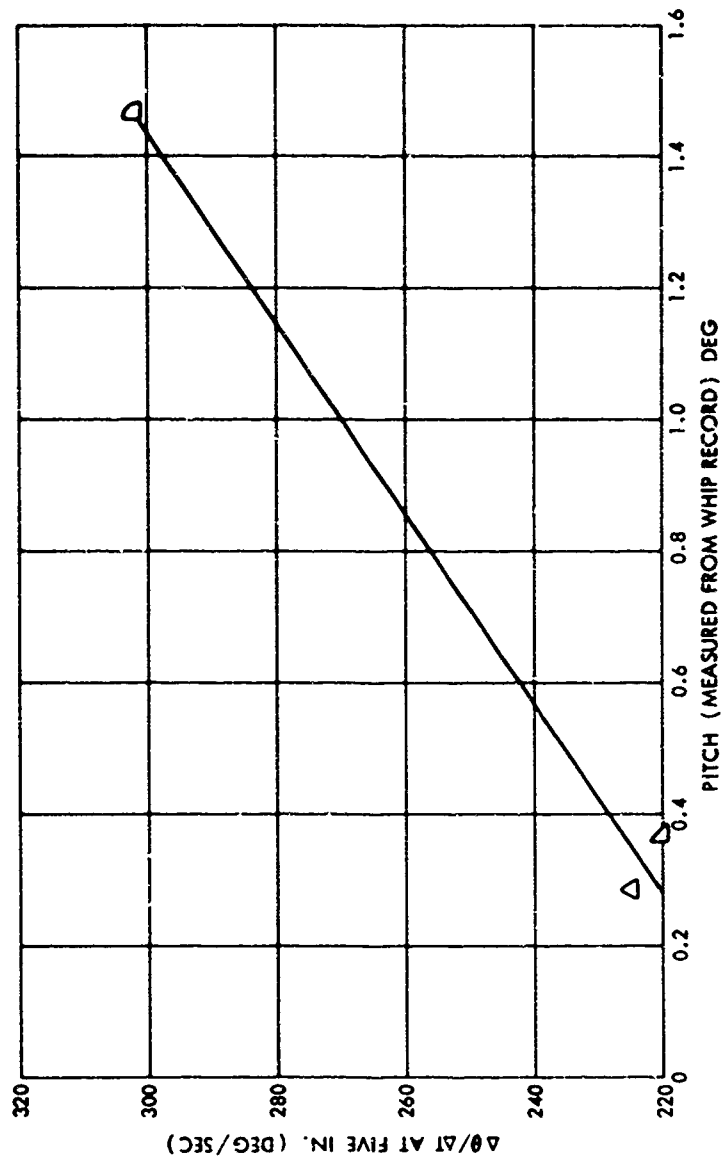


FIG. 17 WHIP VS PITCH (ESTIMATE), MODEL -D

All of the shots reported were reduced for comparison with Equation (7). The first series of shots, in which both CG position and  $I_{CG}$  were allowed to vary, were examined by assuming an average center of pressure on the nose. Assuming that the change in  $\theta$  due to whip is small,  $< 1$  degree, one might conclude that the transient flow field will reproduce from water contact to nose submergence for models of different CG's and  $I_{CG}$ 's, all other variables (i.e.,  $V_0$ , initial pitch, nose shape) being fixed. If this is true, there will be an average lift force,  $\bar{F}$ , and an average center of pressure,  $\bar{X}$ , (figure 1) which will be the same for each model. Thus,

$$\bar{F} (CG + \bar{X}) = I \frac{d\omega}{dt} = I \frac{d\omega}{dt} \frac{ds}{ds} \quad (8)$$

$$\bar{F} (CG + \bar{X}) = I \frac{\Delta\omega}{\Delta s} V_0 \quad (9)$$

$$\bar{F} (CG + \bar{X}) = \frac{I V_0}{s} \Delta\omega \quad (10)$$

The variable parameters are  $I$  and  $\bar{X}$ . A plot of  $\frac{I V_0}{s} \Delta\omega$  vs  $CG$  should be linear with slope  $\bar{F}$  and x-intercept  $\bar{X}$ , both positive. Such a plot is shown in figure 18, with  $\Delta\omega$  taken as  $\Delta\theta/\Delta T$  at

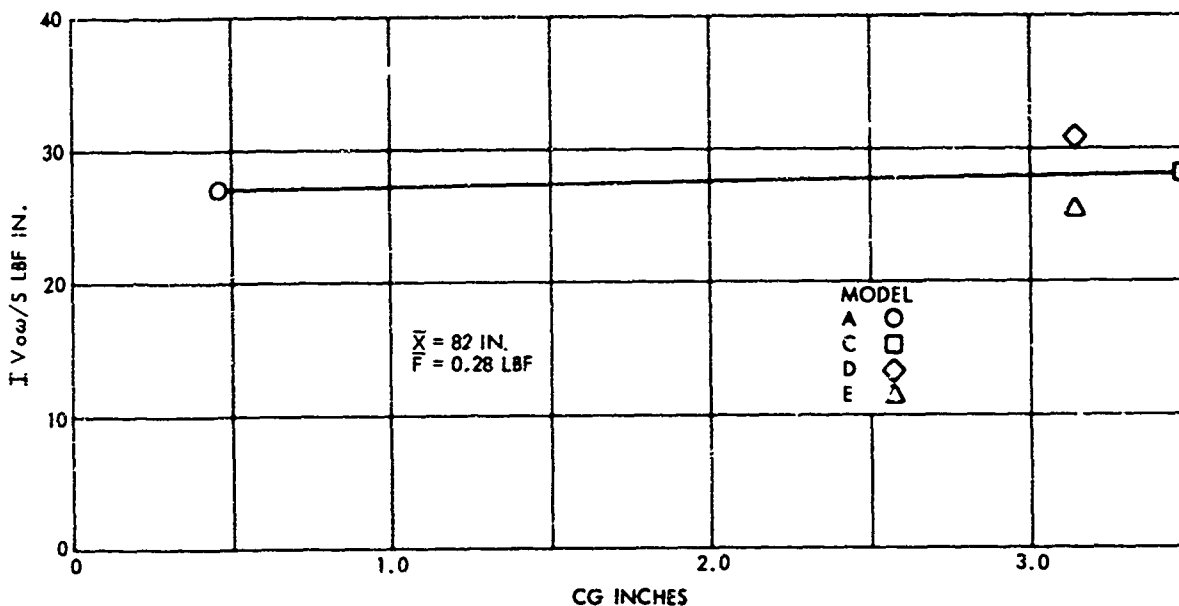


Fig. 18. Plot of Eq. (10)

s = 5 inches and pitch = 0 degree by extrapolation and interpolation in figures 14 and 17. The whip values of models D and E were corrected to  $V_0 = 135$  feet per second by  $\Delta\omega_{135} = \Delta\omega_{105} \cdot (135/105)$  following the assumption that whip is proportional to velocity. The value of  $\bar{X} = 92$  inches is something other than that expected from the theory of a constant pressure on the wetted surface discussed in the Introduction. This graph is, of course, influenced by the assumption that  $\Delta\omega$  varies linearly with  $I_{cg}$  and by the accuracy of the data. (Admittedly, two or three points are bordering on insufficient data. Also, the zero time correlation and the water-entry pitch values were not as precise in these earlier shots as the values listed in Table 2.)

The data taken from the second series of shots (with constant CG position) indicate an almost linear variation of  $\Delta\omega$  with  $I_{cg}$ .  $\Delta\theta/\Delta T$  at depth 5 inches and pitch = 0 degree is plotted in figure 19 for models D and E. Also included in figure 19 is the "whip" corrected to  $V_0 = 105$  feet per second for models A and C. All four points fall very close to the line  $\Delta\theta/\Delta T = CI^{-1.07}$ . There appears to be no dependence on CG position, which supports the conclusion from figure 19 that the time average C.P. is very

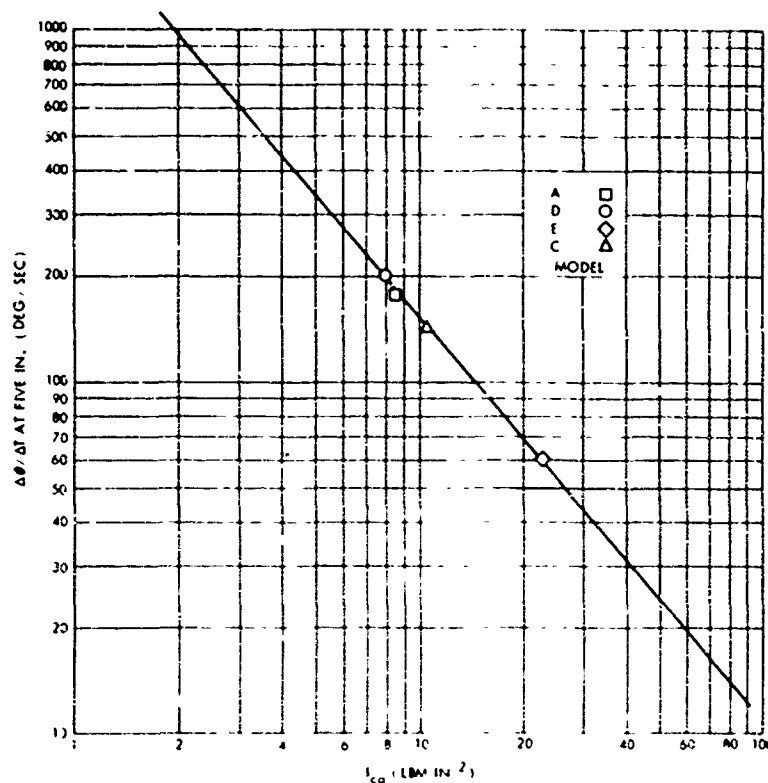


Fig. 19. Whip vs  $I_{cg}$

## NOLTR 66-148

far out in front of the cone tip; essentially, there is a couple acting on the cone. While the results of these two curves are revealing, it is doubtful that the values of the exponent  $-1.07$ ,  $\bar{F}$ , and  $\bar{X}$  should be taken as final. Rather, the matter should be pursued to increase the significance of the data, especially in view of the disagreement with the ideas stated in the Introduction.

### SUGGESTIONS FOR FURTHER INVESTIGATION

One constant cause of trouble in the firing program was the poor launching facility. The scatter in model pitch at water entry exceeded  $\pm 2$  degrees, which was not tolerable since the effective field of view of the whip recorder is less than this, and also the pitch sensitivity of some of the models was so high that they would whip out of the field of view. As a result, most of the shots of models C and D were not reducible. It is believed that reducing the air flight from gun muzzle to water entry (which is presently about 10 feet) will be a necessity for any control over model pitch.

The present line of experimentation should be pursued until either Equation (1) or some other approximation is supported. With improvements in pitch measurement and with a record of  $\Delta\theta/\Delta T$  before water entry, one should be able to have a very clear view of the entire  $\Delta\theta/\Delta T$  vs depth behavior. As it stands now, the beginning portions of the curves (figure 12) are somewhat jumbled and it is only on the latter, fanned out portions that a correlation with initial pitch is possible. If the quality of the curves is improved, one will have a continuous record of the total forces on the nose as it penetrates the water surface.

It appears that the problem of modeling of water-entry whip has been solved experimentally by Waugh (ref. 11), using inertial, Froude, and where needed, gas density modeling. For this reason, there is no immediate need to pursue the matter as a research project. However, these scaling effects must be kept in mind if results from model tests are to be applicable to full-scale missiles.



NOLTR 66-148

#### REFERENCES

1. "Advances in Hydrosience," Ven Te Chow, Academic Press, N. Y., 1964
2. "Impact Forces," Garrett Birkhoff, Applied Mathematics Panel, NDRC (AMP Memo 42.5M), 1944
3. "Summary Technical Report of the Applied Mathematics Panel," NDRC, Vol 1, Mathematics Studies Relating to Military Physical Research, Washington, D. C.
4. "Ricochet Off Water," G. Birkhoff, et al, Applied Mathematics Panel, NDRC (AMP Memo 42.4M), 1944
5. "Head Studies," H. Wayland, (CIT/TLP 28/440), 24 Jan 1945
6. "Optical Method of Measuring Angular Changes of Model Projectiles during Entry Phase," B. H. Rule, (CIT/IPC 69) 17 Mar 1945
7. "The NOL Hydroballistics Facility," H. K. Steves and V. C. D. Dawson, NOLTR 66-125 (in publication)
8. "Approximate Impact Drag Coefficients for the Vertical Water Entry of Families of Cone, Ellipsoidal and Tangent Ogive Nosed Missiles," W. R. Hoover and P. J. Reardon, NOLTR 64-110, 1965
9. "Water Entry Whip and Deceleration of Eight Full-Scale Torpedo Models with Ogive and Spherogive Heads," J. G. Waugh, NOTS, NAVORD Rpt 1223, 2 Mar 1950
10. "First Bureau of Ordnance Hydroballistic Symposium, Water Entry Pitch Modeling," J. G. Waugh, NAVORD Rpt 5793 Confidential, Sep 1957